Math 10 Lecture Videos

Section 5.7:

Negative Exponents and Scientific Notation

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OBJECTIVES:

1. Use the negative exponent rule.
2. Simplify exponential expressions.
3. Convert from scientific notation to decimal notation.
4. Convert from decimal notation to scientific notation.
5. Compute with scientific notation.
6. Solve applied problems with scientific notation.

Objective 1:
Use the negative exponent rule.

The Negative Exponent Rule

If b is any real number other than 0 and n is a natural number, then

$$b^{-n} = \frac{1}{b^n} \quad \text{and} \quad \frac{1}{b^{-n}} = b^n.$$

When a negative number appears as an exponent, switch the position of the base (from numerator to denominator or denominator to numerator) and make the exponent positive.

The sign of the base does not change.

Objective 1:
Use the negative exponent rule.

Examples:

Write x^{-4} with positive exponents. $$x^{-4} = \frac{1}{x^4}$$

Write 5^{-3} with positive exponents. $$5^{-3} = \frac{1}{5^3}$$

Objective 1:
Use the negative exponent rule.

Examples:

$$(-3)^{-4} = \frac{1}{(-3)^4} = \frac{1}{81}$$

$$5^{-3} = \frac{1}{5^3} = \frac{1}{125}$$

$$\frac{1}{7y^{-2}} = \frac{y^2}{7}$$

Objective 2:
Simplify exponential expressions.

An exponential expression is "simplified" when

a. Each base occurs only once.

b. No parentheses appear.

c. No powers are raised to powers.

d. No negative or zero exponents appear.

Objective 2:
Simplify exponential expressions.

SIMPLIFICATION TECHNIQUES	EXAMPLES
If necessary, remove parentheses by using the Products to Powers Rule or the Quotient to Powers Rule.	$(2ab)^4 = 2^4 a^4 b^4$ $\left(\frac{17}{x^2}\right)^3 = \frac{17^3}{(x^2)^3} = \frac{17^3}{x^{2\cdot 3}} = \frac{4913}{x^6}$
If necessary, simplify powers to powers by using the Power Rule.	$\left(Q^{71}\right)^{19} = Q^{71\cdot 19} = Q^{1349}$ $\left(W^5\right)^{10} = W^{5\cdot 10} = W^{50}$

Objective 2:
Simplify exponential expressions.

SIMPLIFICATION TECHNIQUES	EXAMPLES
Be sure each base appears only once in the final form by using the Product Rule or Quotient Rule	$H^4 \cdot H^{16} = H^{4+16} = H^{20}$ $\frac{V^{23}}{V^{17}} = V^{23-17} = V^6$
If necessary, rewrite exponential expressions with zero powers as 1. Furthermore, write the answer with positive exponents by using the Negative Exponent Rule.	$3 + 2\left(45X^3Y^{-4}\right)^0$ $= 3 + 2 \cdot 1 = 5$ $\frac{31}{K^{-12}} = 31K^{12}$

Objective 2:
Simplify exponential expressions.

Examples:

$$x^{-12} \cdot x^{2} = x^{-12+2}$$
$$= x^{-10}$$
$$= \frac{1}{x^{10}}$$

$$\frac{75x^{3}}{5x^{9}} = \frac{75}{5} \cdot \frac{x^{3}}{x^{9}}$$
$$= 15x^{3-9}$$
$$= 15x^{-6}$$
$$= \frac{15}{x^{6}}$$

$$\left(\frac{x^{8}}{x^{4}}\right)^{-5} = \left(x^{4}\right)^{-5}$$
$$= x^{-20}$$
$$= \frac{1}{x^{20}}$$

Scientific Notation

A positive number is written in scientific notation when it is expressed in the form

$$a \times 10^n,$$

where a is a number greater than or equal to 1 and less than 10 ($1 \leq a < 10$) and n is an integer.

Objective 3: Convert from scientific notation to decimal notation.

We can use *n*, the exponent on the 10 in a x 10^n, to change a number in scientific notation to decimal notation. If ***n* is positive**, move the decimal point in *a* to the **right** *n* places. If ***n* is negative**, move the decimal point in *a* to the **left** $|n|$ places.

Objective 3: Convert from scientific notation to decimal notation.

1. Write 7.4×10^9 in decimal notation. The exponent is positive so we move the decimal point nine places to the right.

$$7.4 \times 10^9 = 7,400,000,000$$

2. Write 3.017×10^{-6} in decimal notation. The exponent is negative so we move the decimal point six places to the left.

$$3.017 \times 10^{-6} = 0.000003017$$

Objective 4: Convert from decimal notation to scientific notation

- Determine *a*, the numerical factor. Move the decimal point in the given number to obtain a number greater than or equal to 1 and less than 10.

- Determine *n*, the exponent on 10^n. The absolute value of *n* is the number of places the decimal was moved. The exponent *n* is positive if the given number is great than 10 and negative if the given number is between 0 and 1.

Objective 4: Convert from decimal notation to scientific notation

1. Write $7,410,000,000$ in scientific notation.

$$7,410,000,000 = 7.41 \times 10^{9}$$

2. Write 0.000000092 in scientific notation.

$$0.000000092 = 9.2 \times 10^{-8}$$

Objective 5:
Compute with Scientific Notation

Multiplication

$$(a \times 10^{n}) \times (b \times 10^{m}) = (a \times b) \times 10^{n+m}$$

Add the exponents on 10 and multiply the other parts of the numbers separately.

Objective 5:
Compute with Scientific Notation

Division

$$\frac{a \times 10^{n}}{b \times 10^{m}} = \left(\frac{a}{b}\right) \times 10^{n-m}$$

Subtract the exponents on 10 and divide the other parts of the numbers separately.

Objective 5:
Compute with Scientific Notation

Exponentiation

It's $a^m \times 10^{nm}$ → Correct!

$$(a \times 10^n)^m = a^n \times 10^{nm}$$

Multiply exponents on 10 and raise the other part of the number to the power.

After the computation is completed, the answer may require an adjustment before it is expressed in scientific notation.

Objective 5:
Compute with Scientific Notation

Perform the indicated computation, writing the answer in scientific notation.

$$\left(5.41\times 10^{-6}\right)\left(-3.2\times 10^{-8}\right)$$

$$\left(5.41\times -3.2\right)\times\left(10^{-6}\times 10^{-8}\right)$$ Regroup factors.

$$-17.312\times 10^{-6+-8}$$ Multiply.

$$-17.312\times 10^{-14}$$ Simplify.

$$-1.7312\times 10^{-14+1}$$ Rewrite in scientific notation.

$$-1.7312\times 10^{-13}$$

Objective 5:
Compute with Scientific Notation

Perform the indicated computation, writing the answer in scientific notation.

$$\frac{-3.2\times10^{-8}}{5.41\times10^{-6}}$$

$$\frac{-3.2}{5.41}\times\frac{10^{-8}}{10^{-6}}$$ Regroup factors.

$$-0.59\times10^{-8--6}$$ Multiply.

$$-0.59\times10^{-2}$$ Simplify.

$$-5.9\times10^{-3}$$ Rewrite in scientific notation.

Objective 6: Solve applied problems using scientific notation.

The area of Alaska is approximately 3.66×10^8 acres. The state was purchased in 1867 from Russia for $7.2 million. What price per acre, to the nearest cent, did the United States pay Russia?

We will first write 7.2 million in scientific notation. It is: 7.2×10^6

$$\frac{7.2 \times 10^6}{3.66 \times 10^8}$$

Divide 'dollar amount' by 'acreage'

Objective 6: Solve applied problems using scientific notation.

$$\frac{7.2 \times 10^6}{3.66 \times 10^8}$$ Divide 'dollar amount' by 'acreage'

$$\frac{7.2}{3.66} \times \frac{10^6}{10^8}$$ Simplify.

$$1.97 \times 10^{6-8}$$ Divide.

$$1.97 \times 10^{-2}$$ Subtract.

Therefore, since 1.97×10^{-2} dollars/acre equals \$0.0197/acre, then the price per acre for Alaska was approximately 2 cents per acre.

Objective 6: Solve applied problems using scientific notation.

The cost of President Obama's 2009 economic stimulus package was $787 billion, or dollars. If this cost were evenly divided among every individual in the United States (approximately 3.07×10^8 people), how much would each citizen have to pay?

$$\frac{7.87 \times 10^{11}}{3.07 \times 10^8} = \frac{7.87}{3.07} \times \frac{10^{11}}{10^8} \approx 2.56 \times 10^3 = 2560$$

Each citizen would have to pay about $2560.

OBJECTIVES:

1. Use the negative exponent rule. ✓
2. Simplify exponential expressions. ✓
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5. Compute with scientific notation. ✓
6. Solve applied problems with scientific notation. ✓